

Betrachtung eines Butterworth-TP --- 6.Mai 2007 Ingenieurbüro Baumann, Dorsten

```
• reset():ta:=time():DIGITS:=32:w:=2*PI*f:
```

die Eingangsdaten

```
• n:=4:fg:=10e3:ue2:=1:
```

die Berechnungen

```
• wg:=2*PI*fg:
```

```
• wiwg:=[cos((2*(i-1)+1)*PI/2/n) $ i=1..n]:
```

```
• sigwg :=[-sin((2*(i-1)+1)*PI/2/n) $ i=1..n]:
```

```
• prod:=(f)->product(-sigwg[i]+I*(w/wg-wiwg[i]), i=1..n):
```

```
• U2U0:=(f)->1/(1+1/ue2)/prod(f):
```

```
• U2U0dB:=(f)->20*log(10,abs(U2U0(f))):
```

```
• Winkel:=(f)->-180/PI*sum(arctan((w/wg-wiwg[i])/(-sigwg[i])), i=1..n):
```

```
• Winkel:=(f)->180/PI*arg(U2U0(f)):
```

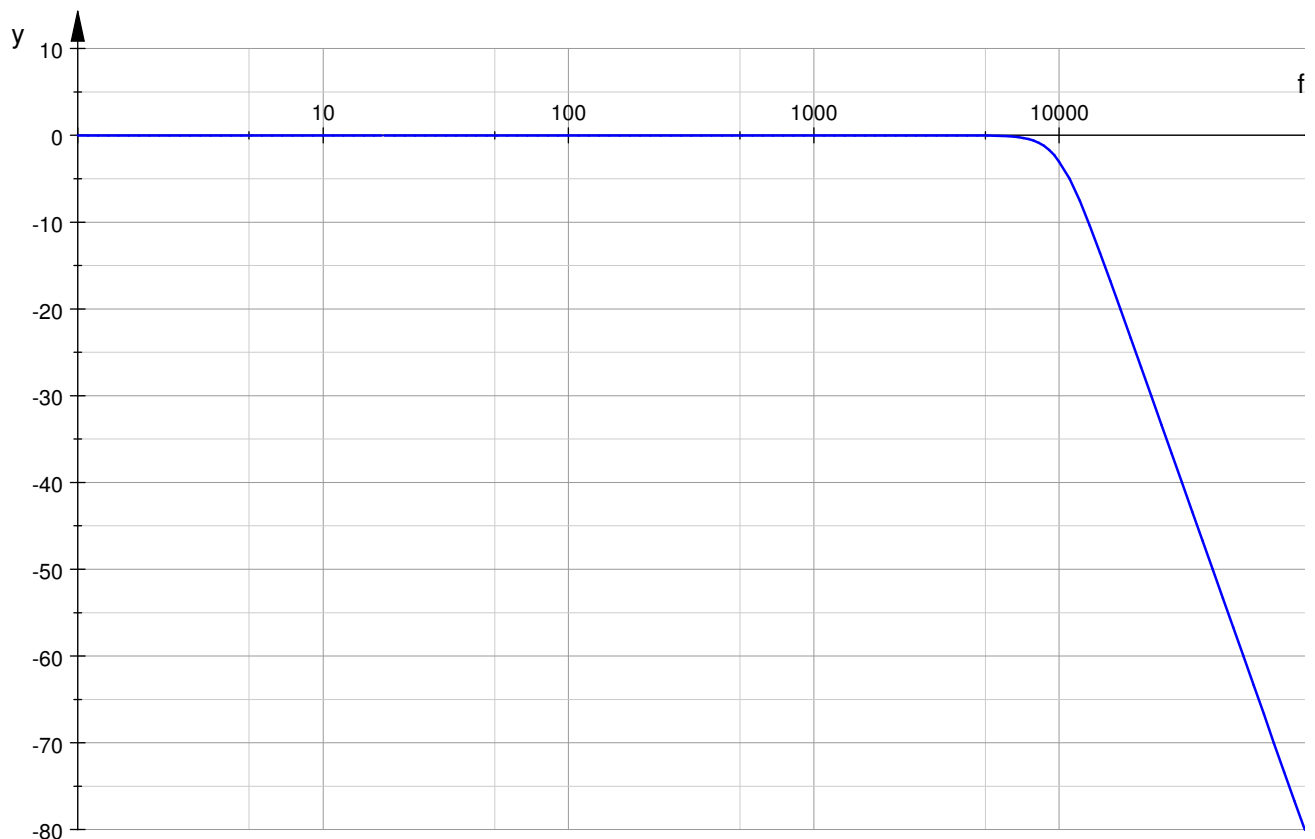
```
• tg:=(f)->-1/wg*sum(sigwg[i]/(sigwg[i]^2+(w/wg-wiwg[i])^2), i=1..n):
```

```
• tg1:=(f)->-diff(Winkel(f),f)/360:
```

Betrag der Übertragungsfunktion, doppelt logarithmisch

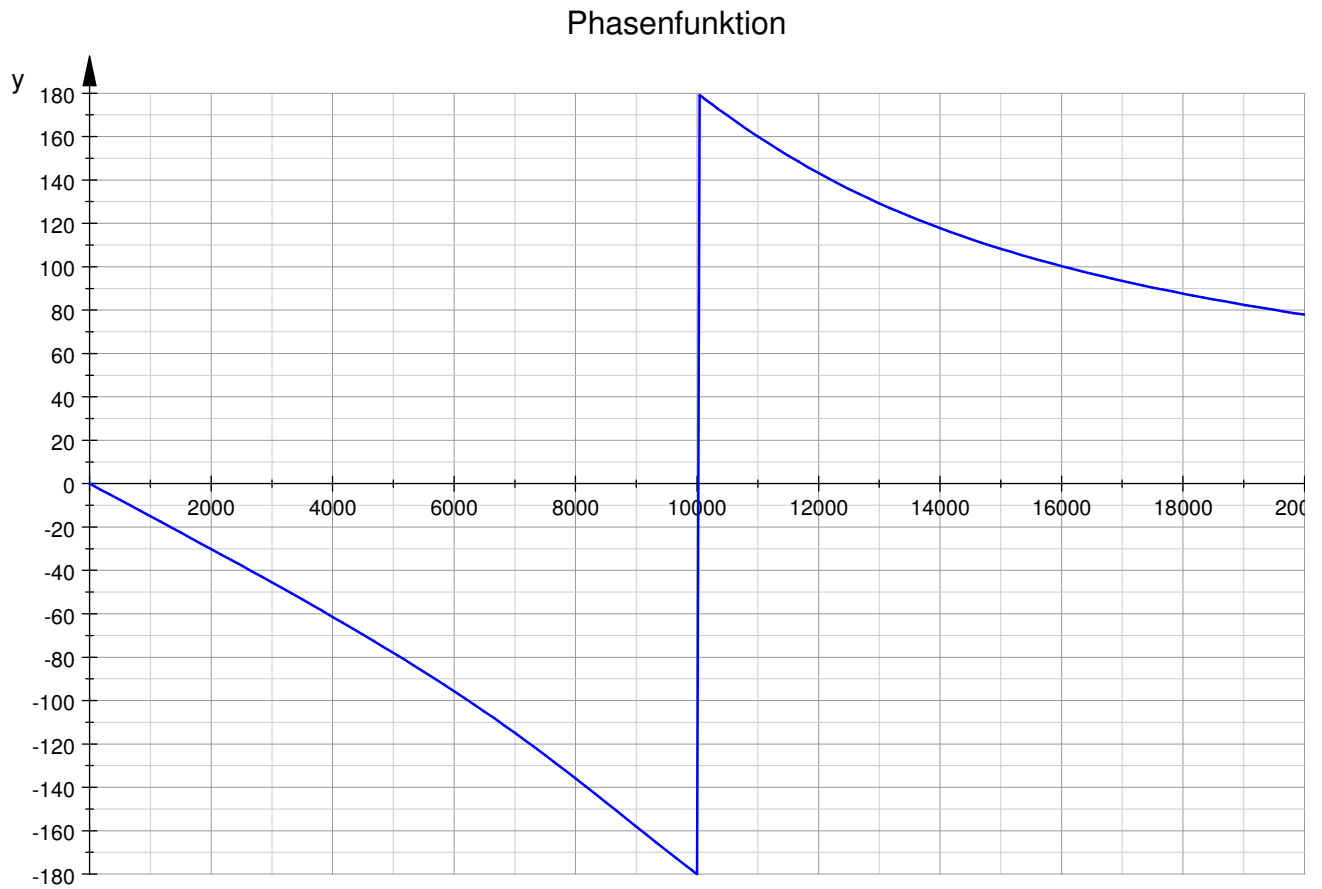
```
• plotfunc2d(U2U0dB(f)+6.01, f=1..10*fg, LegendVisible=FALSE,  
  CoordinateType=LogLin,  
    GridVisible=TRUE, SubgridVisible=TRUE,  
    Height=120*unit::mm, Width=180*unit::mm,  
    Header="Amplitudenfunktion", YMax=10):
```

Amplitudenfunktion



die Phasenverschiebung des Filters

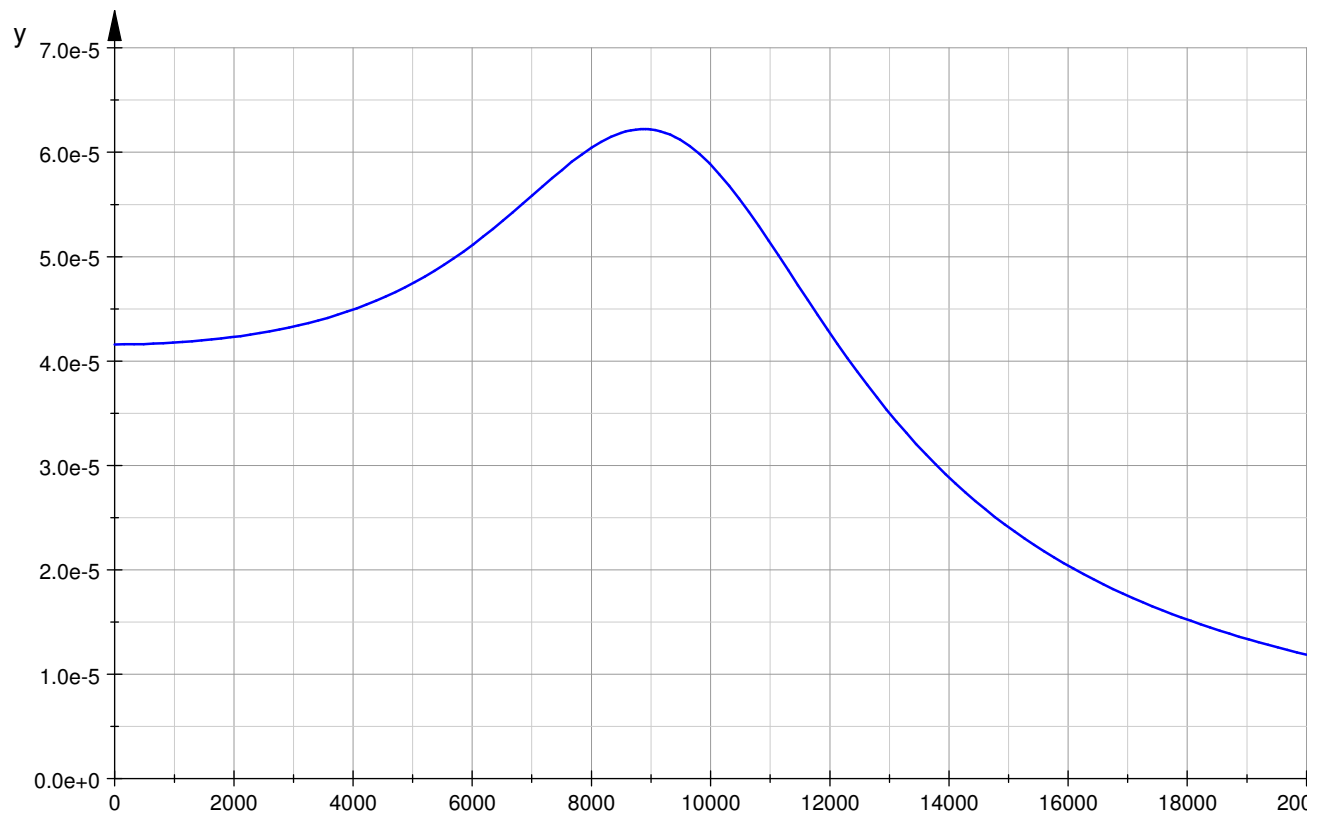
- `plotfunc2d(Winkel(f), f=0..2*fg, LegendVisible=FALSE, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Phasenfunktion"):`



Berechnete Gruppenlaufzeit aus Formel $tg(f)$

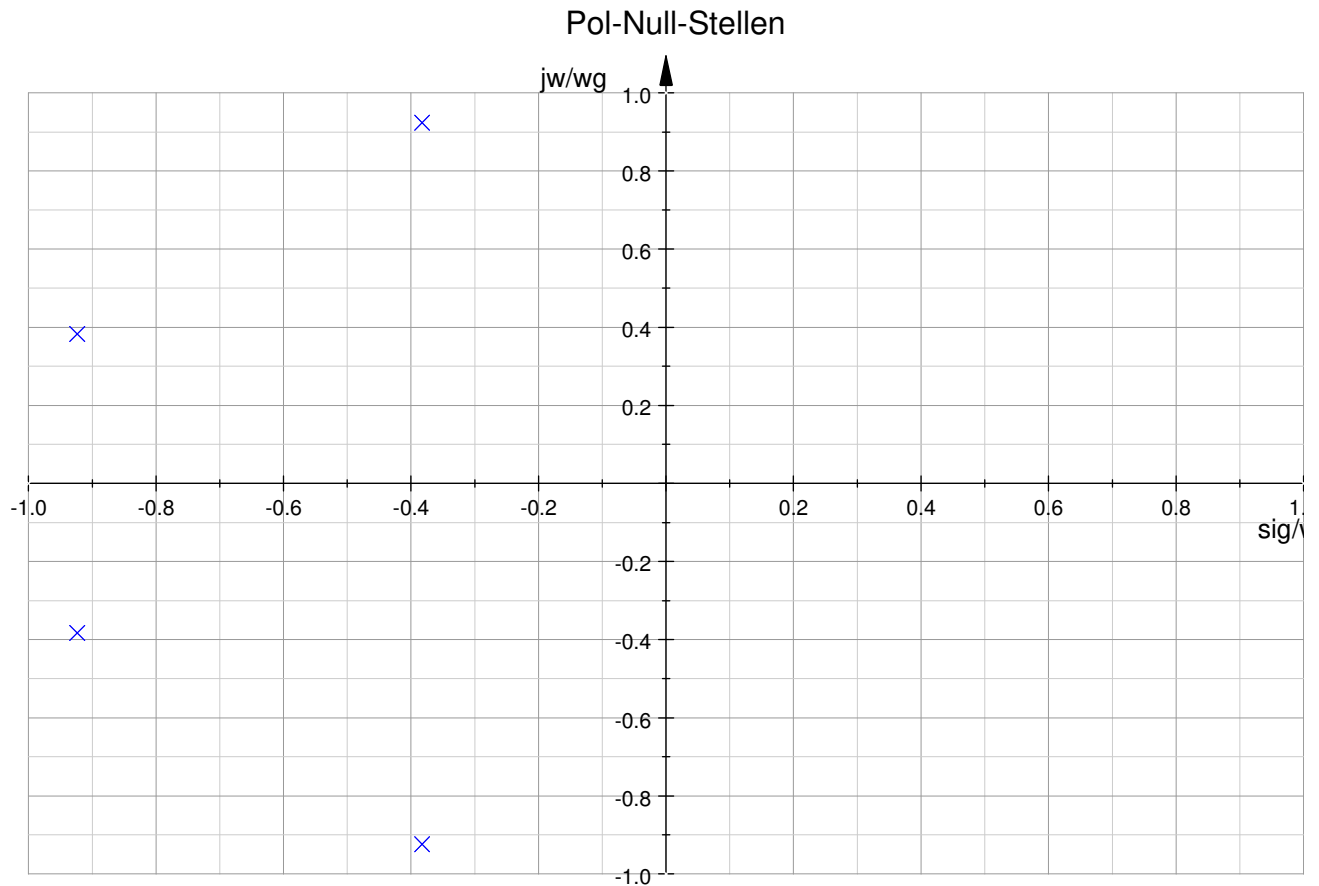
- `plotfunc2d(tg1(f), f=0..2*fg, LegendVisible=FALSE, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Gruppenlaufzeit", YRange=0..7e-5):`

Gruppenlaufzeit



Lage der Pol-Nullstellen in der komplexen Ebene

- `delete Liste:Liste:=[[sigwg[i],wiwg[i],RGB::Blue] $ i=1..n]:`
- `Liste:=Liste.[[1,0,RGB::White]].[[0,1,RGB::White]].[[-1,0,RGB::White]].[[0,-1,RGB::White]]:`
- `plot(plot::PointList2d(Liste, PointStyle=XCrosses, PointSize=2, Color=RGB::Blue, GridVisible=TRUE, SubgridVisible=TRUE, Scaling=Unconstrained, AxesTitles=["sig/wg", "jw/wg"]), Height=120*unit::mm, Width=180*unit::mm, Header="Pol-Null-Stellen"):`



Pol-Nullstellen des Filters

- for i from 1 to n do
 - PolTab[i]:=float(sigwg[i]+I*wiwg[i])
- end_for;
- PolTab;

$$\begin{cases}
 1 & = -0.3826834323650897717284599840304 + 0.92387953251128675612818318939679 \cdot i \\
 2 & = -0.92387953251128675612818318939679 + 0.3826834323650897717284599840304 \cdot i \\
 3 & = -0.92387953251128675612818318939679 - 0.3826834323650897717284599840304 \cdot i \\
 4 & = -0.3826834323650897717284599840304 - 0.92387953251128675612818318939679 \cdot i
 \end{cases}$$

die Pol-Nullstellen durch Suche aus dem Polynom (auch hier:Im u. Re vertauscht und Faktor -1)

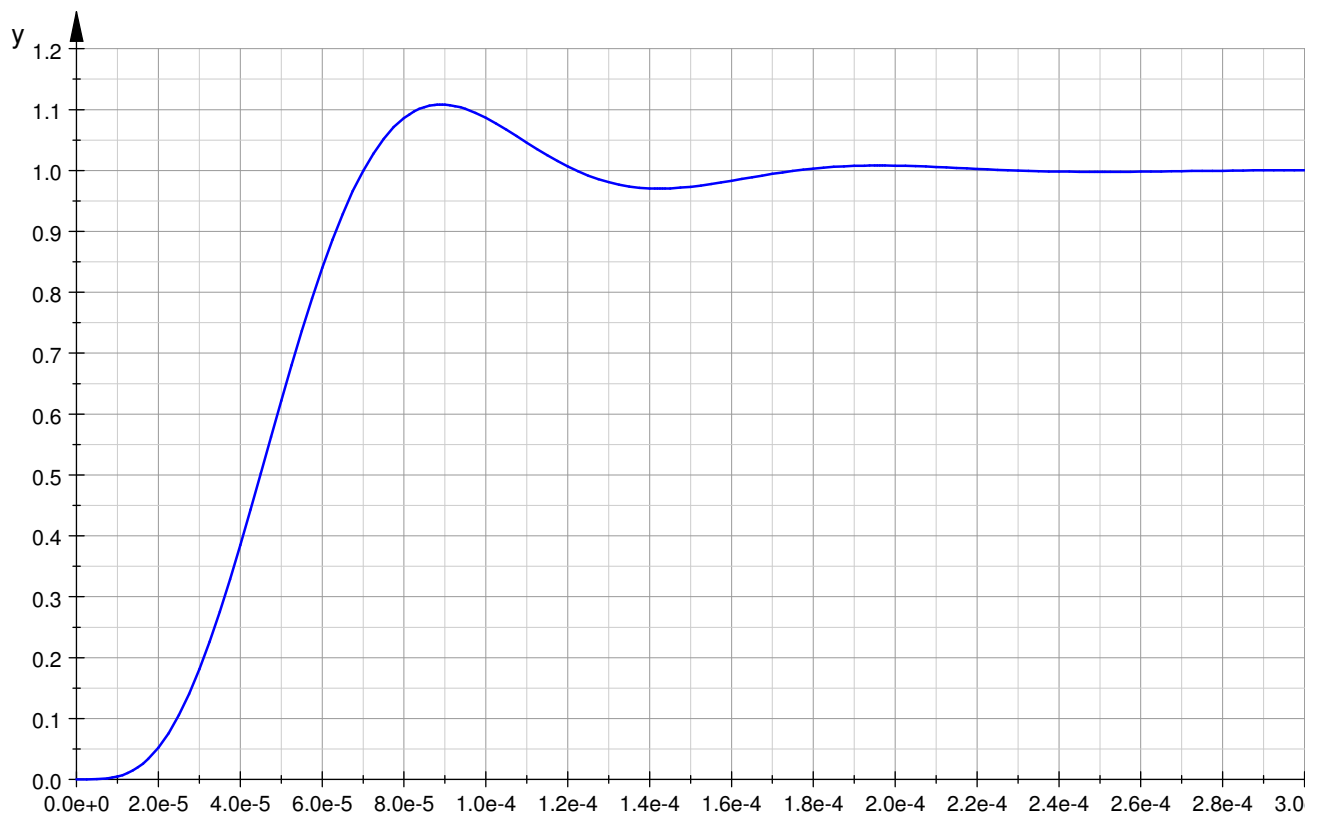
- delete i:Pol:=solve(prod(f)=0, f)/fg:
- for i from 1 to n do
 - PolTabN[i]:=-Im(op(Pol,i))-I*Re(op(Pol,i)):
- end_for;
- PolTabN;

$$\begin{cases} 1 & = -0.92387953251128675612818318939679 + 0.3826834323650897717284599840304 \cdot i \\ 2 & = -0.92387953251128675612818318939679 - 0.3826834323650897717284599840304 \cdot i \\ 3 & = -0.3826834323650897717284599840304 + 0.92387953251128675612818318939679 \cdot i \\ 4 & = -0.3826834323650897717284599840304 - 0.92387953251128675612818318939679 \cdot i \end{cases}$$

Sprungantwort des Filters $ua(t)=\text{invlaplace}(2/p*T(p))$

- `delete i:prodp:=(p)->product(p/wg-sigwg[i]-I*wiwg[i], i=1..n):`
- `ua:=(t)->Re(transform::invlaplace(1/(1+1/ue2)*2/p/prodp(p),p,t)):`
- `plotfunc2d(ua(t), t=0..3/fg, LegendVisible=FALSE, CoordinateType=LinLin, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Sprungantwort", YMax=1.2):`

Sprungantwort



Suchbereich an der Flanke definieren

- `anf:=0:ende:=2/fg:`

Überschwingen in % bei t in us

- `maximum:=op(numeric::solve(diff(ua(t),t)=0,t=anf..ende,RestrictedSearch),1):`
- `(ua(maximum)-1)*100;maximum/1e-6;`

10.830150888537364280420386368286

89.091495433129426852445810912783

t0 für $u_a(t)=1/2$ in μs

- `tx:=op(numeric::solve(Re(ua(t))=1/2,t=anf..maximum,RestrictedSearch),1):tx/1e-6;`

44.885890189741794003857925638927

die Einschwingzeit τ in μs und die daraus resultierende Grenzfrequenz in kHz

- `m:=ua'(t):t:=tx:m:=float(m):delete t:yt:=t->1/2-m*(tx-t):`
- `tau:=op(solve(yt(t)=1,t),1)-op(solve(yt(t)=0,t),1):tau/1e-6;1/2/tau/1e3;`

41.776687025127236519570794894073

11.968397582586365529226696635187

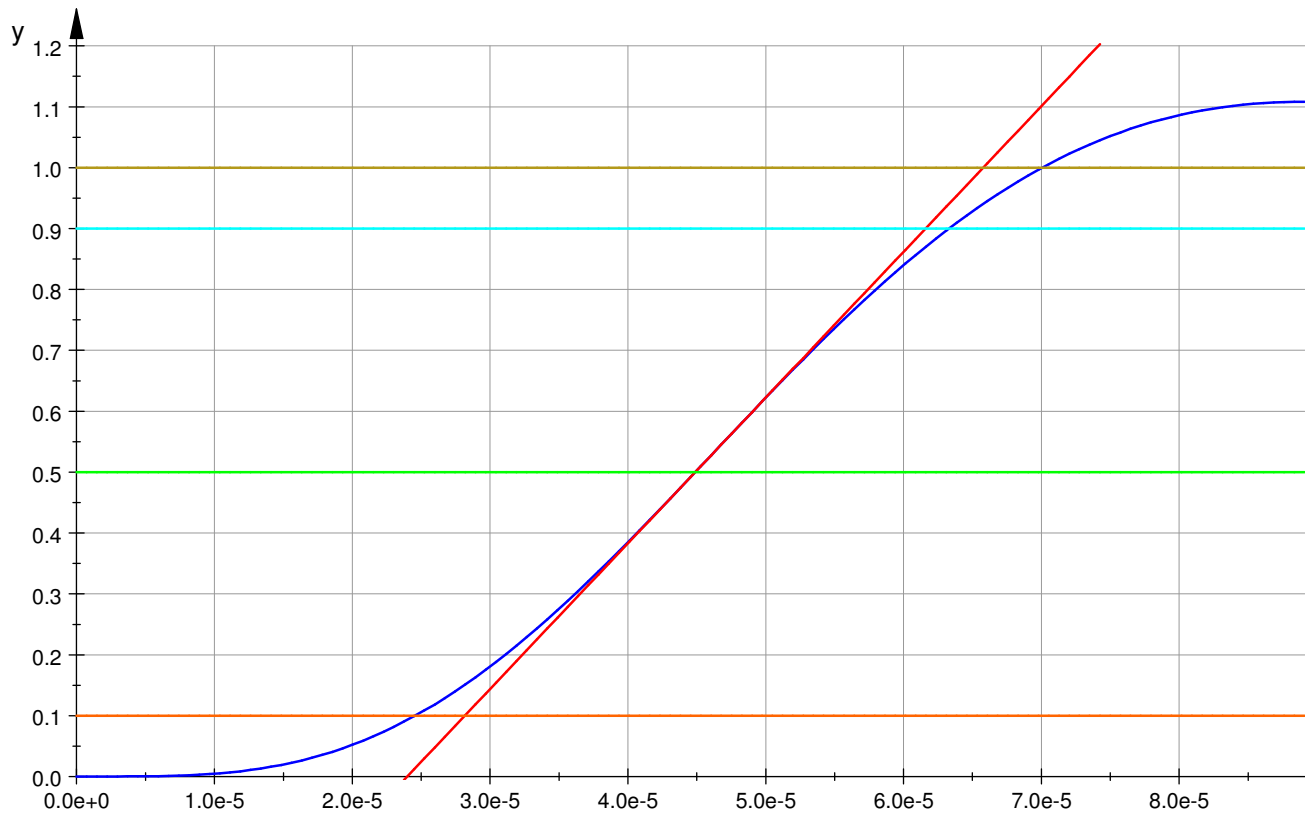
tr, Rise-Time in μs

- `tr:=op(numeric::solve(ua(t)=9/10,t=anf..ende,RestrictedSearch),1)-op(numeric::solve(yt(t)=1/10,t=anf..ende,RestrictedSearch),1):tr/1e-6;`

35.117834672408776683575152053648

- `plotfunc2d(ua(t), yt(t), 1/2, 1, 1/10, 9/10, t=0..maximum,LegendVisible=FALSE, CoordinateType=LinLin,GridVisible=TRUE, SubgridVisible=FALSE,Height=120*unit::mm, Width=180*unit::mm, Header="Vergrößerung Sprungantwort", YRange=0..1.2):`

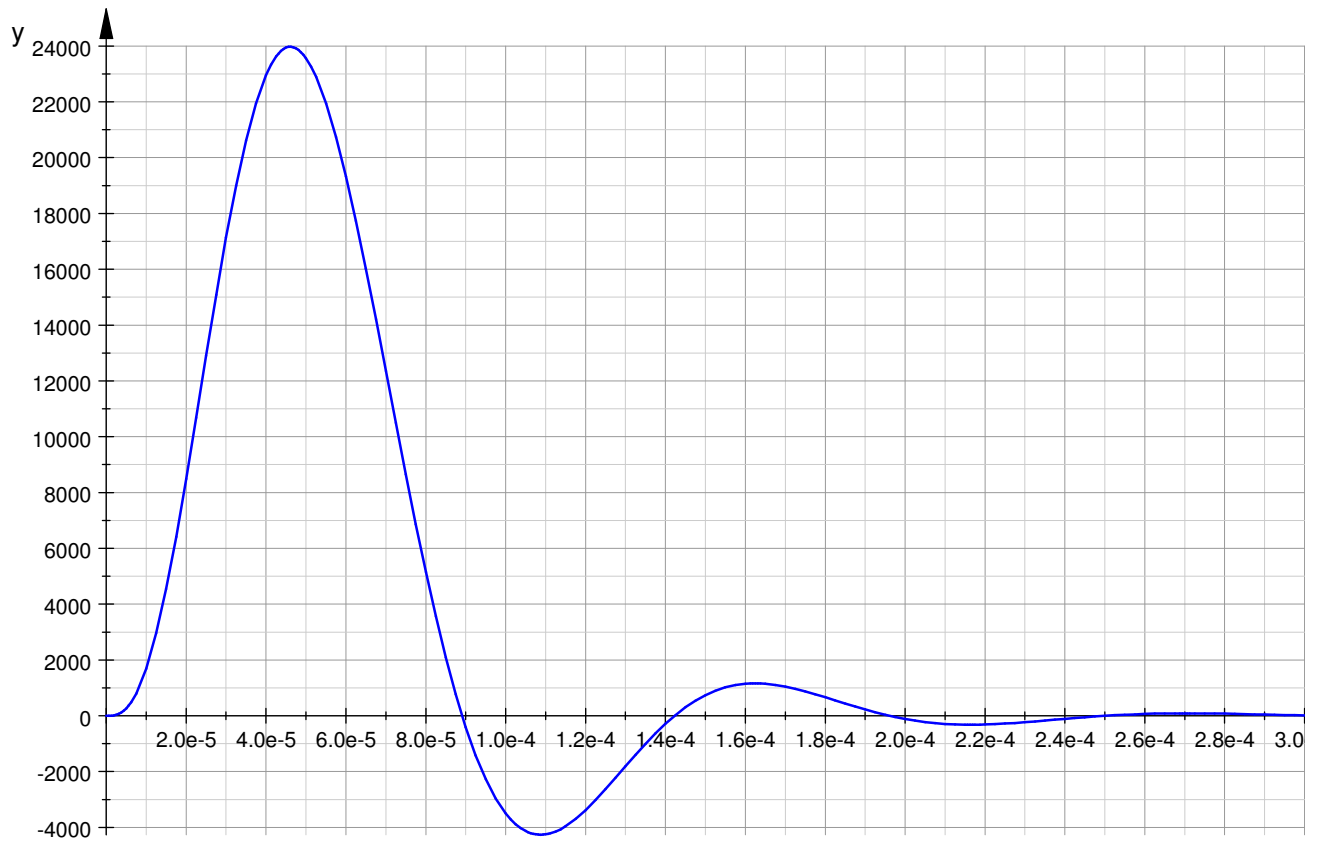
Vergrößerung Sprungantwort



Impulsantwort des Filters $u_a(t) = \text{invlaplace}(T(p))$

- `ua := (t) -> Re(ttransform::invlaplace(1/(1+1/ue2)*2/prodp(p), p, t)) :`
- `plotfunc2d(ua(t), t=0..3/fg, LegendVisible=FALSE, CoordinateType=LinLin, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Impulsantwort") :`

Impulsantwort



CPU-Zeit in Sekunden und in Minuten

- `te:=time():float((te-ta)/1000);float((te-ta)/1000/60);`

1.719

0.02865

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