RG213U

```
reset():ta:=time():DIGITS:=48:c0:=299792458:Z0:=50:Z1:=50:Z2:=100:
l:=100:x:=50:Cs:=101.049872e-12:Rs:=6.56167979e-3:
```

Prozeduren: Numerische, inverse Laplace-Transformation (Talbot), für t nicht 0 eingeben

Talbot suggested that the Bromwich line be deformed into a contour that begins
and ends in the left half plane, i.e., z ? -8 at both ends.
Due to the exponential factor the integrand decays rapidly
on such a contour. In such situations the trapezoidal rule converge
extraordinarily rapidly.
For example here we compute the inverse transform of F(s) = 1/(s+1) at t = 1
#---
#Octave:
#>> pkg load symbolic
#>> syms s
#>> F=1/(s+1)
#F = (sym)
#
1
-----
s + 1
#
#>> error=talbot(function_handle(F),1,24)-exp(-1)
#ans =   1.6098e-015
#---
#
Talbot method is very powerful here we see an error of 1.61e-015
with only 24 function evaluations
#
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Date : 25 October 2009
#
Reference
L.N.Trefethen, J.A.C.Weideman, and T.Schmelzer. Talbot quadratures
and rational approximations. BIT. Numerical Mathematics,
46(3):653 670, 2006.
#
Shift contour to the right in case there is a pole on the positive real axis : Note the contour will
not be optimal since it was originally devoloped for function with
singularities on the negative real axis
For example take F(s) = 1/(s-1), it has a pole at s = 1, the contour needs to be shifted with one
unit, i.e shift = 1. But in the test example no shifting is necessary

```
Talbot:=proc(F_s, t, N)
local h,shift,ans,theta,k,z,dz;
begin
 h:=2*PI/N;
 shift:=0;
 ans:=0;
 for k from 0 to N do
   theta:=-PI+(k+1/2)*h;
   z:=shift+N/t*(0.5017*theta*cot(0.6407*theta)-
  0.6122+0.2645*I*theta);
   dz:=N/t*(-
```

```
    0.5017*0.6407*theta/sin(0.6407*theta)^2+0.5017*cot(0.6407*theta
    )+0.2645*I);
     ans:=ans+exp(z*t)*F_s(z)*dz;
   end_for:
     return (Re((h/(2*I*PI))*ans));
  end_proc:
```

Induktivitätsbelag in uH/m

- `Ls:=Z0^2*Cs:float(Ls/1e-6);`

$$0.25262468$$

Ableitungsbelag in uS/m

- `Gs:=Rs*Cs/Ls:float(Gs/1e-6);`

$$2.624671916$$

Ausbreitungsgeschwindigkeit auf der Leitung in m/s

- `vl:=1/sqrt(Ls*Cs);`

$$197922071.588571631243629878125921822048423772373$$

Verhältnis Ausbreitungsgeschw. / Lichtgeschw.

- `vl/c0;`

$$0.66019696729185772659974614213250762315182649582 4$$

Laufzeit für x Meter in us (2 Methoden)

- `td:=x/vl:float(td/1e-6),float(x*sqrt(Ls*Cs)/1e-6);`

$$0.25262468,\ 0.25262468$$

Übertragungsfunktion der Leitung

- `gam:=sqrt((Rs+p*Ls)*(Gs+p*Cs)):`
- `/* Tp:=(Z2*cosh(gam*(l-x))+Z0*sinh(gam*(l-x)))/((Z1+Z2)*cosh(gam*l)+(Z0+Z1*Z2/Z0)*sinh(gam*l)): */`

sinh() u. cosh() umformen in e-Funktionen

- `a:=gam*(l-x):b:=gam*l:`
- `Tp1:=(Z2*(exp(a)+exp(-a))+Z0*(exp(a)-exp(-a)))/((Z1+Z2)*(exp(b)+exp(-b))+(Z0+Z1*Z2/Z0)*(exp(b)-exp(-b))):`

Erregung Sprungfunktion 1/p

- `lap:=(p)-->expand(1/p*Tp1);`

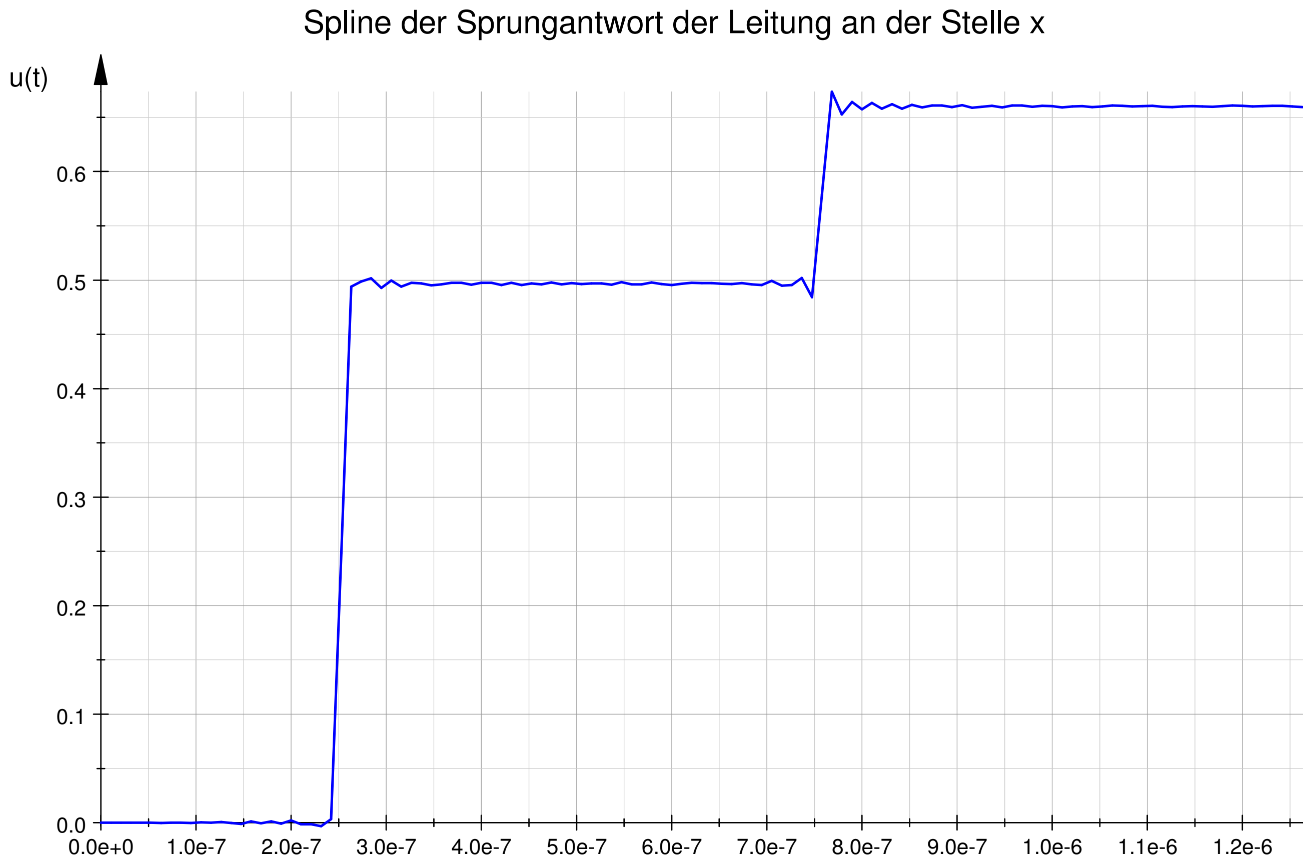
$$p \rightarrow \frac{1}{2 \cdot p \cdot \left(e^{\sqrt{0.000000000000000025527691578040 96 \cdot p^2 + 0.00000000000132611380576897376 \cdot p + 0.000000017222256666597\ldots}}\right.}$$

Anzahl der Stützstellen u. Talbot-Iterationen

- `M:=200:Talits:=768:`
- `Liste:=[[float(5*td/M*i), float(Talbot(lap,5*td/M*i,Talits))] $ i=1..M]:`
- `plot(plot::PointList2d(Liste, PointStyle=Crosses, PointSize=1, GridVisible=TRUE, SubgridVisible=TRUE,`

```
                    Scaling=Unconstrained,
  AxesTitles=["t","u(t)"]), Height=120*unit::mm,
  Width=180*unit::mm, Header="Sprungantwort der Leitung an der
  Stelle x"):
```

- `x:=array(0..M-1, [float(5*td/M*(i+1)) $ i=0..M-1]):`
- `S:=numeric::cubicSpline([x[i], float(Talbot(lap,x[i],Talits))] $ i=0..M-1, Natural):`
- ```
  delete x:plot(plot::Function2d(S(x), x=0..5*td), GridVisible=TRUE,
  SubgridVisible=TRUE,
                      Scaling=Unconstrained,
    AxesTitles=["t","u(t)"], Height=120*unit::mm,
    Width=180*unit::mm, Header="Spline der Sprungantwort der
    Leitung an der Stelle x"):
  ```
  ```



Berechnung eines Funktionswertes

- `S(8e-7);`

0.657297739311355343226498816852638052419212515709

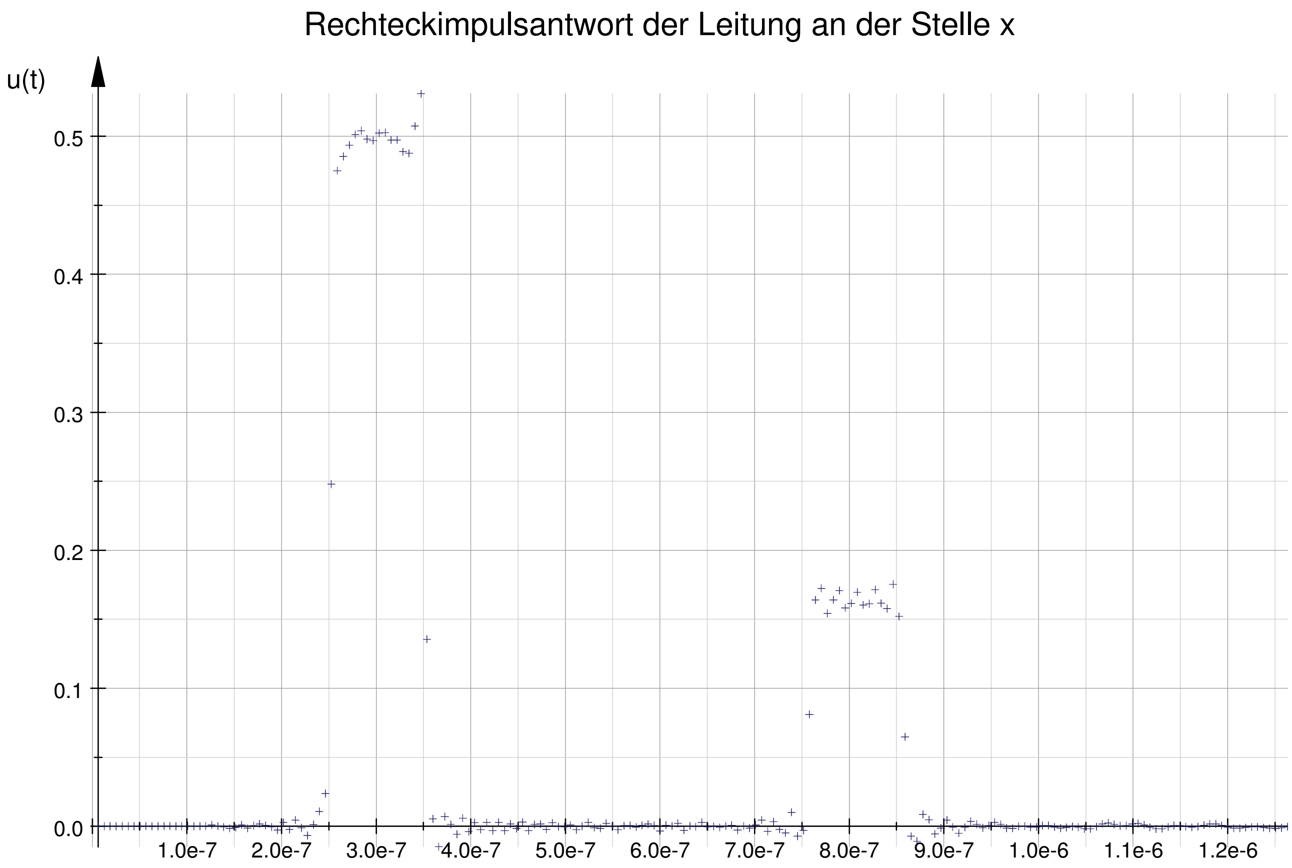
Erregung Rechteckimpuls 0.1 us

- `Ug1:=1/p*(1-exp(-1e-7*p)):`
- `lap:=(p)-->expand(Ug1*Tp1);`

```
p -> 1/2/p/exp((0.00000000000000002552769157804096*p^2 +
0.000000000001326113805768973 76*p +
0.00000001722225666659777764)^(1/2))^50 +
1/6/p/exp((0.00000000000000002552769157804096*p^2 +
0.000000000001326113 80\
576897376*p + 0.00000001722225666659777764)^(1/2))^150 -
1/2/p*exp(-0.0000001*p)/exp((0.00000000000000002552769157804096*p^2
+ 0.000000000001326113805768973 76*p +
0.00000001722225666659777764)^(1/2))^50 - 1/6/\
p*exp(-0.0000001*p)/exp((0.00000000000000002552769157804096*p^2 +
0.00000000000132611380576 89\
7376*p + 0.00000001722225666659777764)^(1/2))^150
```

- `delete Liste:Liste:=[[float(5*td/M*i), float(Talbot(lap,5*td/M*i,Talits))] $ i=1..M]:`

```
plot(plot::PointList2d(Liste, PointStyle=Crosses, PointSize=1,
GridVisible=TRUE, SubgridVisible=TRUE,
                       Scaling=Unconstrained,
  AxesTitles=["t","u(t)"]), Height=120*unit::mm,
  Width=180*unit::mm, Header="Rechteckimpulsantwort der Leitung
  an der Stelle x"):
```




```
x:=array(0..M-1, [float(5*td/M*(i+1)) $ i=0..M-1]):
```

```
S:=numeric::cubicSpline([x[i], float(Talbot(lap,x[i],Talits))] $
i=0..M-1, Natural):
```

```
delete x:plot(plot::Function2d(S(x), x=0..5*td), GridVisible=TRUE,
SubgridVisible=TRUE,
                       Scaling=Unconstrained,
  AxesTitles=["t","u(t)"], Height=120*unit::mm,
  Width=180*unit::mm, Header="Spline der Rechteckimpulsantwort
  der Leitung an der Stelle x"):
```

Berechnung eines Funktionswertes

```
S(8e-7);
```

0.1584748947376978660707099387796387778225771834 38

CPU-Zeit in Sekunden und Minuten

```
float((time()-ta)/1e3);float((time()-ta)/1e3/60);
```

23718.016

395.30026666666666666666666666666666666666666667

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